Epicycles as a Math Perturbation Technique

Epicycles accounted for the motion of a planet's dot of light about the sky. The dot moves along circles that themselves move around larger circles. Start with the largest circle that almost describes the motion, then add the next smaller, then the next. Not just any circles, just those precisely *so*. Modern folks think this is silly, it sounds clumsy and arbitrary. To me it sounds like an early application of math perturbation methods.

Perturbation Theory A set of mathematical steps that are used for ultra-complicated models like quantum electrodynamics (QED) and the very successful Standard Model of quantum particles.

(1) The variables can be written as a large value and a smaller add-on, call it $\boldsymbol{a}$.
We would prefer $\boldsymbol{a}$ to be at least 1/10 of the "large" part, sometimes it must be bigger.

(2) Rewrite master equations using Large+Small form for the variables.

(3) Expand all terms to get a polynomial in $\boldsymbol{a}^{k}$, each term successively smaller

Zero order solution. Simplify the main equations by dropping all small terms, that is, all terms involving $\boldsymbol{a}$. It is a simple value, and not exactly right. But it is close, it is calculable, and that is its advantage.
First order correction – Go back to step 3. Now keep the term in $\boldsymbol{a}$, but drop all terms with $\boldsymbol{a}^2$, $\boldsymbol{a}^3$, … . The term you keep this time is the zeroth order with a small-valued correction. Not perfect, but in better agreement with the true but uncalculatable value.
Second order correction, – Go back to step 3, again. Now keep all terms up to $\boldsymbol{a}^2$, but drop all the ones with higher exponents. See? Your value is better yet!

Each cycle is more complicated and harder to do, but follow perfectly logical steps. Stop when fatigued. The results can be in extremely accurate agreement with the best data. Sometimes this even works when the "small" parts of the variable are not so tiny. Perturbation techniques work for quantum calculations and are respectable, *even though it is known that if you carry the expansion too far your answer will start to* diverge – *approach infinity*.

I see epicycles to be early early efforts to do an expansion in small parameters, before they people understood the math that was to be used … without the *automatic logic* of modern perturbation techniques.

0th order solution: Planets (from ancient Greek: *wandering stars*) rotate about Earth and also move against the stellar background in the same direction but faster that those "fixed" background stars.

Note, the planets are just bright lights in the sky

Adjustable Constants – 2:

R	Main (average)distance between Earth and planet
ω °/wk	Main rotation constant about the Earth, measured in degrees/week (**°/wk**) This is the Greek symbol "omega."

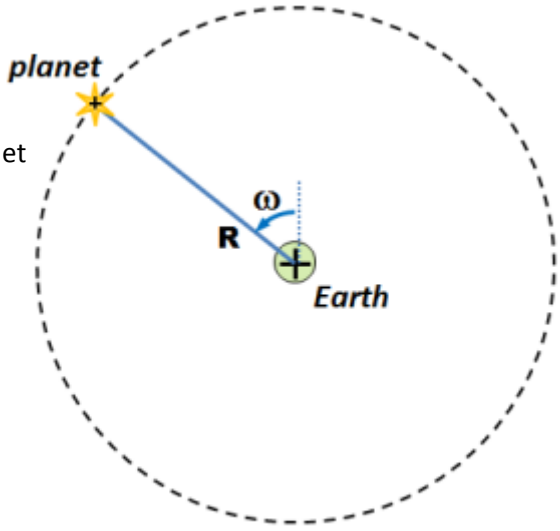

Fig 1: *Zeroth order solution*
Everything rotates about Earth

Epicycles as a Math Perturbation Technique

1st order correction: Some planets, like Venus, cross the sky in the same direction as the common stream of background stars (prograde motion), but some, like Mars occasionally reverse direction for a while.

With the sun as the center of motion, Earth sees the inner planets to always move in the prograde direction; the outer planets will experience both prograde and retrograde motion, Mars shows the largest effects.

Hipparchus (*Greek*, about 150 BC) corrected the 0th solution to account for retrograde motion by having the planet rotate in circles (epicycles) about a point that moves along the 0-th model's path.

Adjustable Constants – 4 total

R	Main (average) radius.
ω °/wk	Main rotation of epicenter about the Earth.
ρ	The epicycle radius, the distance between *planet-epicenter* (gr. symbol "rho").
α °/wk	rotation speed around the epicenter (gr symbol "alpha"). You are free to select **α** as either clockwise or counterclockwise motion.

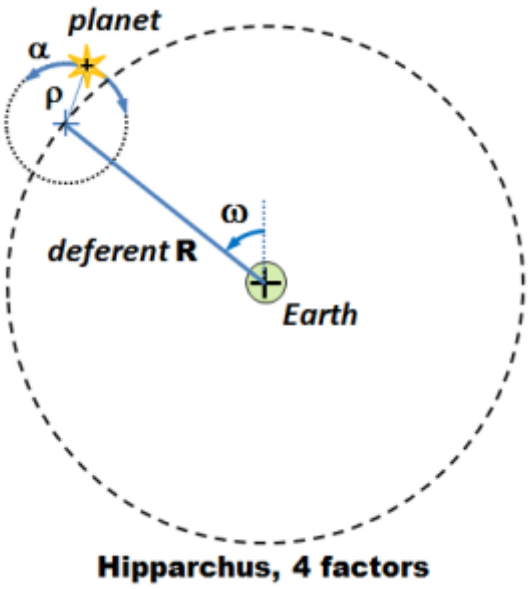

Fig 2 *First order solution*: epicycles correct for retrograde

2nd order correction: Planets actually change their speed as they move in the orbit, fastest when they are closest to the sun, .

Ptolemy (*Greco-Egyptian* about 150 AD) added an offset Earth to the center of the main rotation (Earth-eccentric) actually to account for the Earth-Sun distance. Later, to get the non constant speed, he added another distance (Earth-equant) to the point where the epicenter rotates about the system at the constant angular velocity.

Adjustable Constants – 6 total

R	Radius, now called the *deferent*, from planet epicenter to an offset point some distance from Earth, called the *eccentric*.
ω °/wk	Main rotation speed of epicenter about a point (the equant) further from Earth than the *eccentric*, but on the same line to the Earth.
ρ	The epicycle radius
α °/wk	rotation speed around the epicenter
E_C	The *eccentric* offset.
E_Q	The *equant* offset

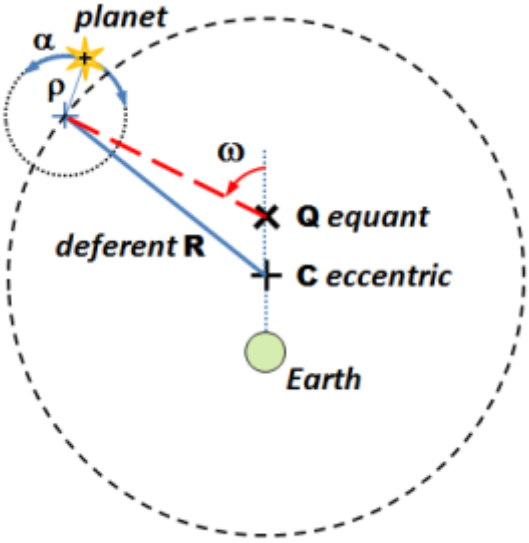

Fig 3 *Second order solution*: first correction for speed

Epicycles as a Math Perturbation Technique

Each correction was a “perturbation” because it added small values to the previous solution. Even this was not quite accurate because the planets actually orbit the sun in (slightly) elliptical orbits.

Later corrections added sequences of ever-smaller epicycles to better approximate the ellipse. Though they never realized they were approaching the shape of a solar-centric ellipse, the old guys used math *automatic logic* to predict planetary locations, and their numbers matched their then-current data.

The 2, 4 and 6 guessed-at parameters in the epicycle models look a lot like modern computer models, from nuclear bomb modeling (LASNEX) to all the weather codes currently in operation. They each contain physics modules that make the predictions, and a bunch of “free” (meaning guessed at) parameters to represent things either not known, or too complicated for detailed modeling.

Epicycle models were not laughable, they were steps forward along the path to modern science!

Fifteen hundred years later, Tyco Brahe’s astonishingly good data provided the evidence that something fundamental was wrong with the theory. Johannes Kepler organized it, and Isaac Newton devised a new explanation that produced a highly accurate model and finally changed astrology into modern astronomy.

This did not mean the Ptolemy’s epicycles were “wrong,” anymore than Newton’s gravity-based “Laws” were proved “wrong” by Relativity. Both were good for their time, both were replaced by a new model that agreed everything the previous did, but were simpler, made new predictions, and had better accuracy.

Actually this is how Science marches forwards.

This document is a PDF file located here.

It first appeared as an Appendix to our post Physics – Science or Ideology?.

Charles J. Armentrout – Ann Arbor

LastTechAge.com