Estimating Reservoir Depletion

This document is located at http://lasttechage.wordpress.com/pdf-references/

Symbols

m = number of regular withdrawals per year. Let T be the number of years for the total of N ... $N = m\,T$

A_1 = the amount withdrawn during first cycle of the draw-down. Clock starts.

r = would be annual rate of increase in withdrawals if there were only 1 withdrawal each year (m=1).

i = fraction of increase of current withdrawal last amount withdrawn. By definition: $i = r/m$ also $m\,i = r$

Y_0 = expected lifetime of reservoir (in years) ... assuming that every amount removed were the same as A_1.

Q_0 = Size of Reservoir before withdrawals, in units appropriate to what is being stored.
If the same amount A_1 is withdrawn each year: $Q_0 = (m\,A_1)\,Y_0$ = (amount taken each year) × Years

f = total currently lost from reservoir. ($f = 0.3$ means reservoir is 30% depleted, 70% remaining)
Start (Q=0): $f = 0$, End ($Q = Q_0$): $f = 1$ and reservoir has been totally depleted. Q is total withdrawn.

During cycle 1 withdraw A_1

During cycle 2 a bit more $A_2 = A_1 + i \cdot A_1 = A_1(1+i)$

During cycle 3 Take a bit more $A_3 = A_2 + i \cdot A_2 = A_1(1+i)^2$

During cycle 4 Take a bit more $A_4 = A_3 + i \cdot A_3 = A_1(1+i)^3$

Generalize from cycles 1 – 4: After N withdrawals, $A_N = A_1\,(1+i)^{N-1}$.

Q (total removed) is $Q = A_1 + A_2 + \ldots + A_N$ Apply geometric sequence formula .... $Q = A_1 \dfrac{(1+i)^N - 1}{i}$

Total quantity removed $Q = f\,Q_0$, substitute .... $f\,Q_0 = A_1 \dfrac{(1+i)^N - 1}{i}$

since $Q_0 = m\,A_1\,Y_0$, we get: $Q = f\,m\,A_1 Y_0$ substitute and simplify ($r = i \cdot m$) .... $r\,f\,Y_0 = (1+i)^N - 1$

Since $N = mT$ we get 0........ $(1 + r f\,Y_0) = (1 + r/m)^{mT}$

Find time T when reservoir holds $Q = f\,Q_0$ Withdraw m times per year
Solve last equation for T, time to deplete reservoir .... $T = \dfrac{1}{m}\,\dfrac{\ln(1+r f\,Y_0)}{\ln(1+r/m)}$, same as
(recall that $1/m = i/r = (1/r) * i$) $T = \dfrac{1}{r}\,\ln(1 + r f\,Y_0) \cdot \left(\dfrac{i}{\ln(1+i)}\right)$

Continuous drain:

Set up the condition as when contents drain continuously,
as water in a bucket with a small hole ... let the number cycles per year grow without bound: $m \to \infty$.

- Make last factor in gray box continuous: set $\rho = m/r$ ... $(1 + r/m)^{mT} = (1 + \frac{1}{\rho})^{\rho\, rT} \xrightarrow[\rho \to \infty]{} e^{r \cdot T}$

- The term $(1+\frac{1}{\rho})^{\rho}$ turns into the "natural" base $e \approx 2.718$ .... $T = \frac{1}{r}\ln(1 + r f\,Y_0)$
  Result is $(1 + r f\,Y_0) = (1 + r/m)^{mT}$, solve for T. (Easy to use with a calculator.)

Suppose we find an oil resource that will last 200 years if drained at a constant rate. If usage rises 3%/year, how long until it becomes 1/2 empty ($f = 0.5$)? Use the continuous formula because oil is pumped "continuously"

$T = \ln(1 + 0.03 * 0.5 * 200)/.03 = 46$ years. ... *Be careful* on a calculator using "(" and ")"

How long will reservoir last if production remains exponential until the last drop is pumped?

$T = \ln(1+0.03 * 1 * 200)/.03 = 64.8$ yrs

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M. King Hubbert Proposal:

Growth to satisfy demand cannot be maintained much past the 50% depletion point.
Peak in resource use comes at a time close to this half-empty time.

After the peak, price will rise substantially, extraction will become technically intense, ultimately value not worth cost.

In 1971* Hubbert estimated 150-300 years of world oil both found and estimated.

We should expect peak to occur sometime around 2010, *assuming oil usage has grown 5% each year on the average*.

For resource that lasts at Y_0 = 225 yr, 5% usage growth rate: T = ln(1+.05*0.5*225)/0.05 about 38 yrs, 1972+38=2010

Error interval: Y_0 = 150 yr, T_{PK} = 31 yr … 2003
Y_0 = 300 yr, T_{PK} = 43 yr … 2014

Error handling was not done very well, here.
Personally, I expect to see world peak to be 2007–2020.

Hubbert emphasized that peak will not be visible until long after event. This will be highly political …

Depletion Model used:

A math model is needed to make any estimate on how long a reservoir will last when it is under active depletion
This is described in LastTechAge report: *Exponential Growth and Depletion Of A Reserve.pdf*

Fig 1: Depletion of most reservoirs follows a production curve similar to this curve, although the shape after the peak might vary a lot. The golden years of exponential growth are on the left side of the curve, and there comes a time (at the arrow) when extraction becomes too difficult to maintain the original exuberant growth. There is nothing to requres any particular shape after exponentiation, and the curve is fragile, easily changed in curvature, and might even become multiple peaks, depending on local politics.

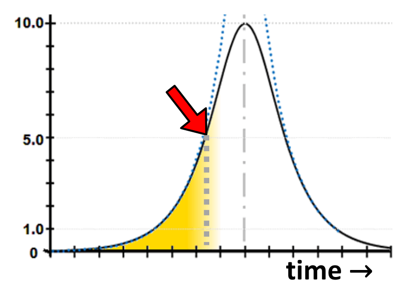
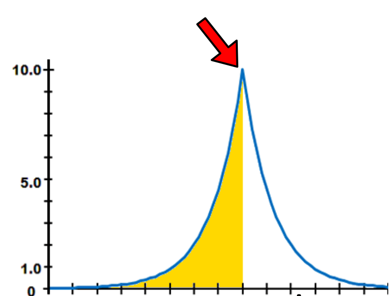


Fig 2: Our work here follows the simplified model shown. We assume that golden exponentail growth is true right up to the time of the peak, itself. Although this "cartoon" plot shows an exponential decay in production after the peak, we do not really care what that shape is.

The model used for the time-to-peak estimate is the simple Hubbert assumption: When 1/2 the reservoir has been drained, extraction becomes much more difficult and production must drop. This means that the peak of production is very near the 1/2 useage point.



simple model used
for quick estimates

Common sense tell us that after the real peak, prices will rise. For a required material such as oil, the demand will not drop by much. This means that production can bring in expensive higher technology and continue on. This would indicate that the post-peak time is when prices go ever upward, production might slow a bit, but never actually stop.