Modeling Depletion Of A Reserve

This document is located at http://lasttechage.wordpress.com/pdf-references/

Stages of Growth – How to tell when exponential production ends.

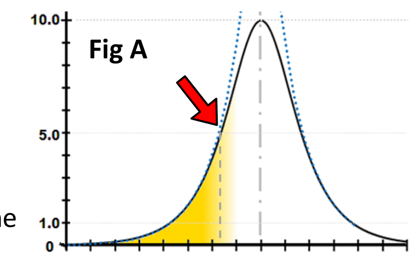
Removal of the resource stored in a reservoir can be considered a "process;" and might refer to pumping petroleum from an oil field, reducing the population of bacteria on a Petri dish (reservoir=collection of all bacteria on the plate), or sales growth of a company (reservoir=total sales for the product). When growth of a "process" has no *external* forcing controls, its own internal processes will control its rate of growth. This action is universal and is about the most understandable self-organizing pattern there is.

(A) As production happens, new users appear who want what current users have, and new applications appear as users learn more about the product.

(B) Next cycle: because demand rose last cycle, more is made. The amount Demand and production increase next cycle is proportional to how much is produced this cycle.

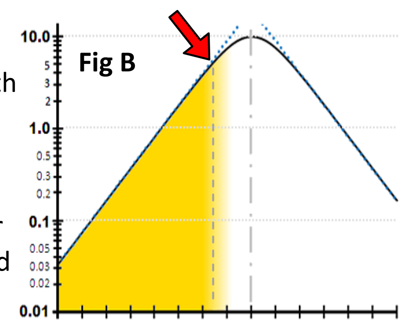
(C) Return to step (A)

New growth is proportional to the current size of the *process*. The graph is the dramatically upward curving line called *exponential*. **Fig A: left dotted blue line is exponential *growth*; right line is exponential *decay*.**



Growth of most processes goes through 3 standard stages:

1. **Startup Time:** key feature: *Exponential* growth. If *process* is a small company, these are the **golden years**. Every year, the process expands as a fraction (or percent) of last year's process.
2. **Transition Time:** key feature: process bends away from exponential growth. This stage starts at the time marked by the arrow. Ultimately, the process reaches a peak then heads downwards.
3. **Post-Peak Time:** This is usually modeled as an exponential decay of the process and begins after the peak has passed. This is reasonable if the "process" is, for example, a bactericide spray. Such processes have growth curves that are symmetrical; the peak happens when half the reservoir is used.



Our technique here – We use a logarithmic (log) graph of data, as in Fig B, for the same data as Fig A. Exponential curves are straight lines on Log plots, and it is easy to see when this exuberant startup stage ends (see arrow). Find details on how this works in *Visualize Exponential Growth.pdf* .

Fig A is a standard "Cartesian" plot, called a lin-lin graph because the steps are linear – all the same size; ours goes from 0 to 10, steps of 1. This shows good images near the peak, but not for data smaller than 1/10th of maximum.

Fig B is a log-lin plot (see box). The signature are the bands that are a factor of 10 wide. B has 3 such bands from 10 to 1, from 1 to 0.1, and from 0.1 to 0.01. The steps are not uniform but squeeze together, moving from up from the bottom of each band.

Log-Linear Graphs

- Plot the logarithm of the data on a Cartesian plot, but label the vertical y axis with the actual data values.
- Straight lines indicate exponential data.
- The rate of increase for exponential processes (oil extraction processes measured in bpd) is proportional to the current process size. Exponential *growth rates* are percents of the current data size, not constant amounts.
- Data fluctuations lie closer to trendlines than in linear graphs. This makes treads easier to see.

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Details On The Algebra Model

The concept of "Peak Oil" stems from the behavior we discussed. M. King Hubbert was one of the first to use a model to explore the implications of the production peak the occurs during depletion of a reservoir.

Hubbert Function: D-Logistic

Hubbert needed a function that would start up with a pure exponential curve, then somewhere roll over to a peak value. For simplicity, he wanted the function to be symmetric about the peak. This allowed the observation that the peak of resource production will occur when the reservoir is half empty. He selected the calculus derivative of the logistics. Although non-obvious, it met all his needs.

Key Feature1: Production changes from exponential to transition when production ≈ peak /5
Key Feature2: if *no* transition region, exponential crosses peak time at exponential value ≈ 4× peak

Reciprocal hyperbolic cosine function: R-Cosh

This also meets all the requirements, and is easier to calculate. There are some small differences between these two model functions.

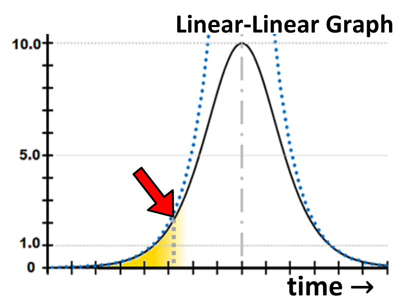
Key feature1: break between exponential and transition production ≈ peak/2
Key Feature2: if *no* transition region, exponential crosses peak time at exponential value ≈ 2× peak

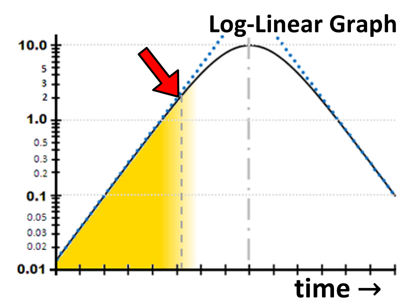
This change in production time is the main difference between the two models. The R-Cosh model keeps break point closest to the peak. To help visualization on the last page, our model was the R-Cosh.

Hubbert – derivative of logistics curve

$$\frac{e^{-t}}{(1+e^{-t})^2} = \frac{1}{4\cosh^2(t/2)}$$

exponential → transition y ≈ Peak/5

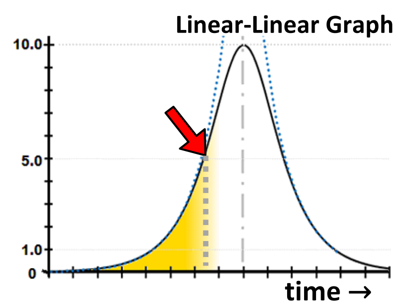


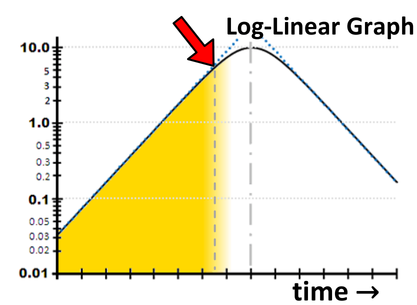


Alternate – Reciprocal of cosh t

$$\frac{1}{\cosh(t)} = \frac{2}{e^{t}+e^{-t}}$$

exponential → transition y ≈ Peak/2

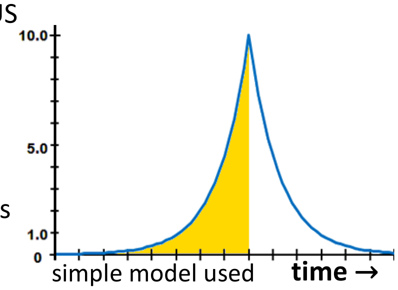




Some oil reservoirs seem to follow one model, some follow times. The US followed Hubbert's curve. Venezuela maintained exponential growth closer to peak time than the 'Alternate' model.

A quick formula was developed to estimate when the peak should occur using a simplified model – no transition time; depletion is a golden exponential up to the time of the peak. These estimates are lower bounds to the occurrence of the depletion peak

See LastTechAge Age *Estimating Reservoir Depletion.pdf*



simple model used for quick estimates

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