

# Suddenness of exponential growth hard to grasp

## The Chess Board's Tale

### Kingdom is ravaged by terrible beast

- destroys crops, children,
- villagers, taxpayers.

*King* sends out plea for help  
*Brave Young Man* responds

**KING:** Save us, I will give *anything* in reward

**BYM:** Ok, my reward will be your daughter's hand in marriage

**KING:** Yes, yes, anything

Month later, *Kingdom saved!* Victorious **BYM** returns bloody,  
limping, arm in sling ... ready for his new wife

**King reneges:** About that daughter thing, sorry, Dude!

Changed my mind – Rich old count down the way just offered *great* dowry

**BYM:** Well, ok, I withdraw my claim if you do this...

Day 1: 1 penny on square 1 of chess board,

Day 2: 2 pennies on second square,

Day 3: 4 pennies on third,

Each successive day, 2× previous day.

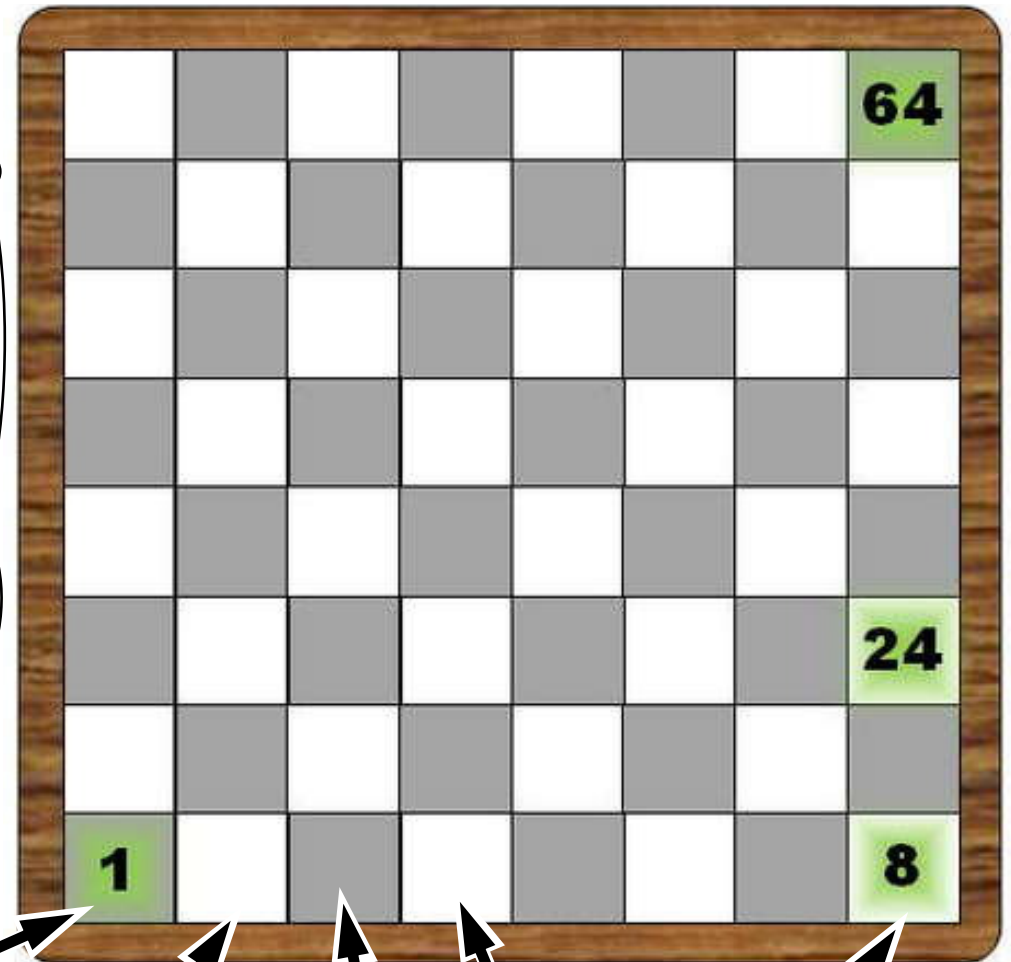
Ancient  
story

# Each square, 2× the number of pennies

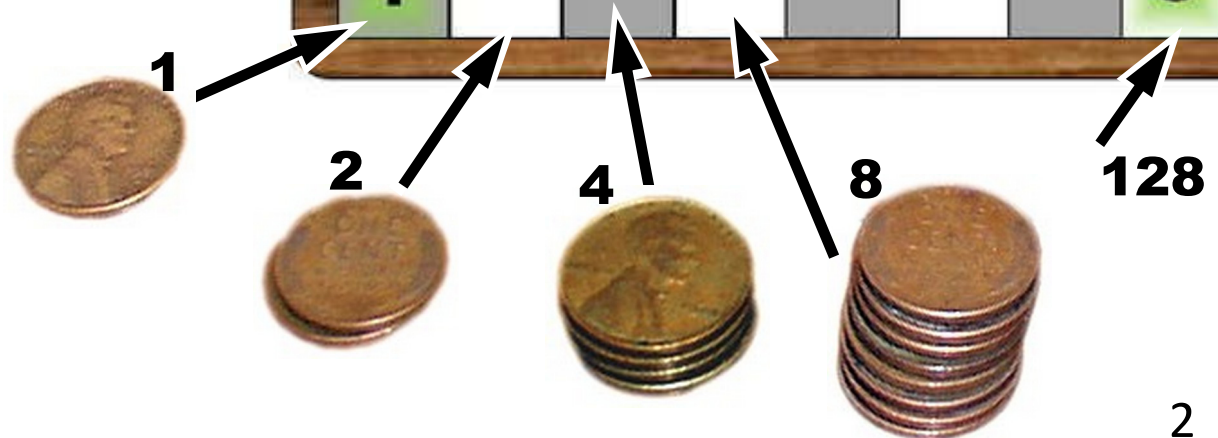
## King cogitates

"Hmm —  
4th day, only  $1+2+4+8 = 15$  pennies  
Square 8... *"I can count that"* 128  
Square 16 ... +8 squares  $8 \times 128$  1024  
Square 24 probably  $8 \times 1024$  8192  
Square 64 ... 268,435,456 ¢  
only  $\approx \$2.70$  M?

*"Is BYM stupid, or ...  
could I have made a mistake?"*



*Actually, our King is  
even worse with math  
than he is with running  
his kingdom!*



# Days 5,6 7 playing exponential growth game

The King saw process:

- Tomorrow's gain in height is always proportional to the stack's height today.

**Tomorrow** —  
the stack will be twice as tall as it is **today**.

Hey! No problem.

*Tomorrow:* Only 128¢ 7"

*Day after?* 256¢ 14"

- The royal treasury has truly *huge* resources. Tomorrow's demand will be trivial to meet, as well as the next day's!



"This is easy, surely we can keep going *forever*."

# On day 8, 128 pennies were placed on square 8



If you double each step – square 16 will be 150 feet high ! *Really?*

## King needed expert help

Call in ...

*the **Court Mathematician***

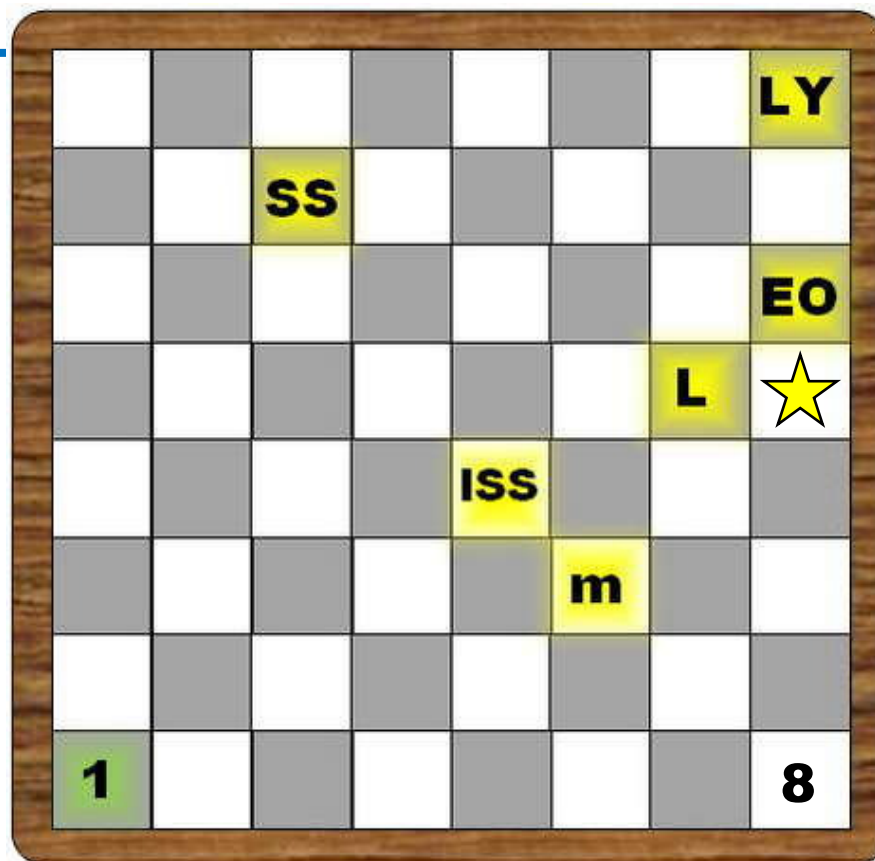
The CM ponders, measures, calculates

*penny 1.4 mm thick (0.055"),  
... daily doubling ...*

By Day 22 he was ready to explain

Here is his final diagram →

The game had reached square 22 (**m**)  
the stack towered more than **1 mile**  
with  $2^{21} = 2,097,152$  coins.



*Tomorrow, the coin stack will be twice as high.*

CM's chart of the game

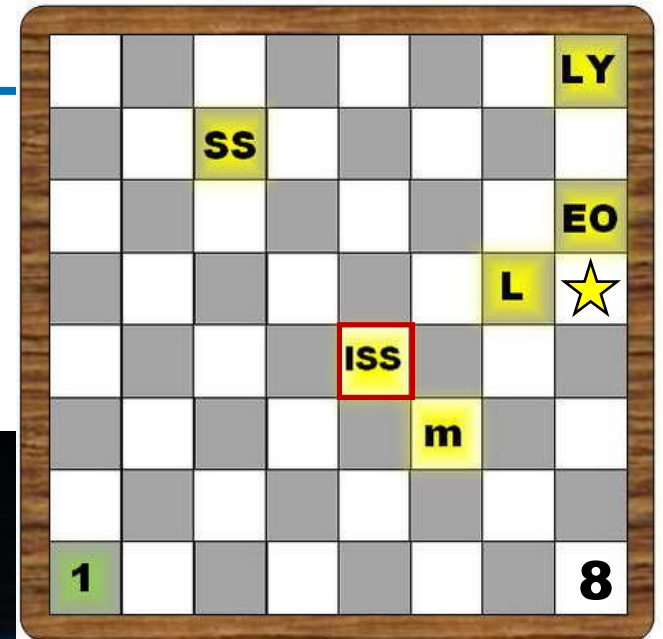
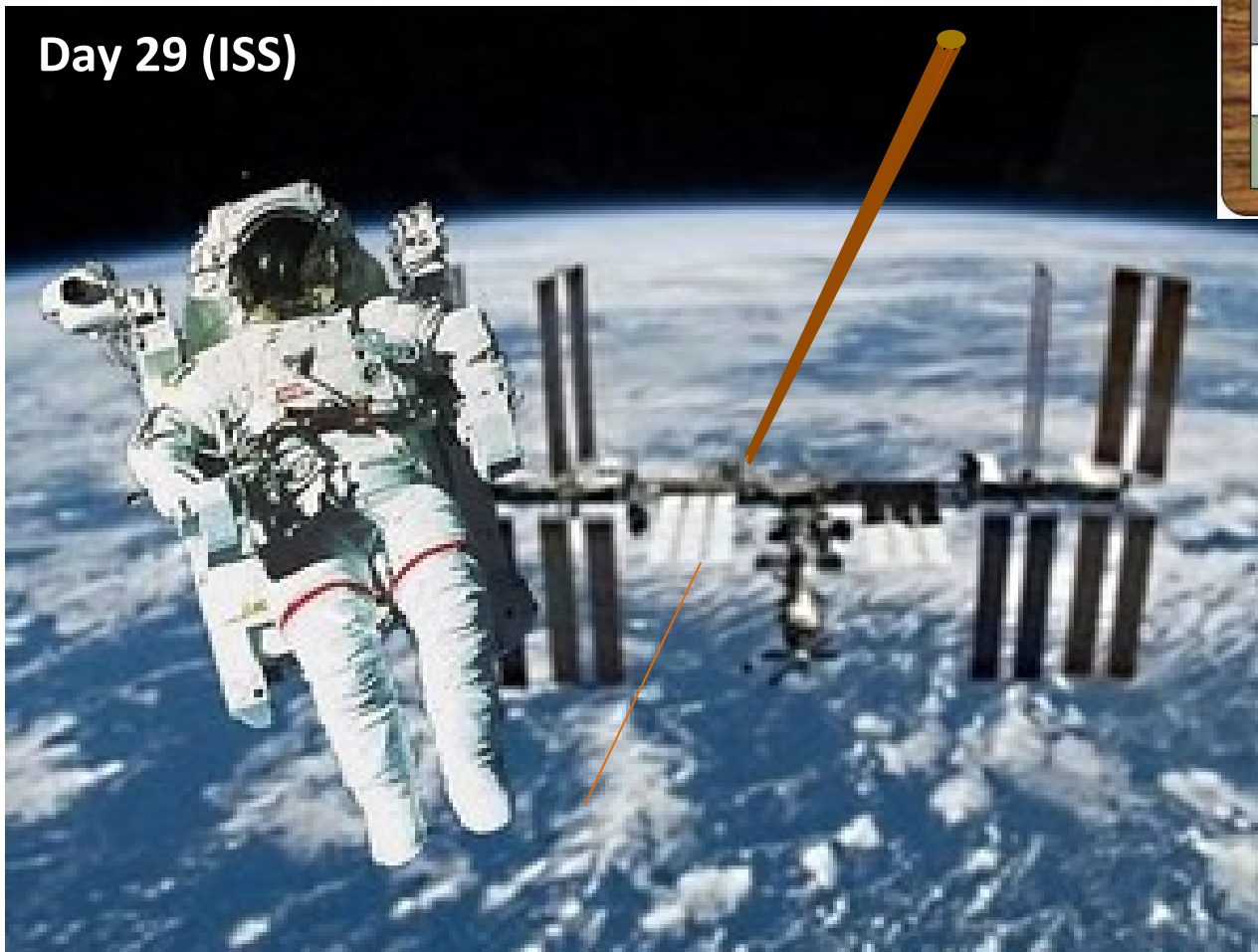
**No outside constraints to growth:**

**CM:** "M'lord, let us assume that progress is *limited only by the game rules* and we can, build a new tower each and every day. Here is what will happen ... "

# The CM's story - 1

**Day 29** Enjoyn' a quiet orbit about the Station. Suddenly a coppery spike shoots from Earth and zips past.

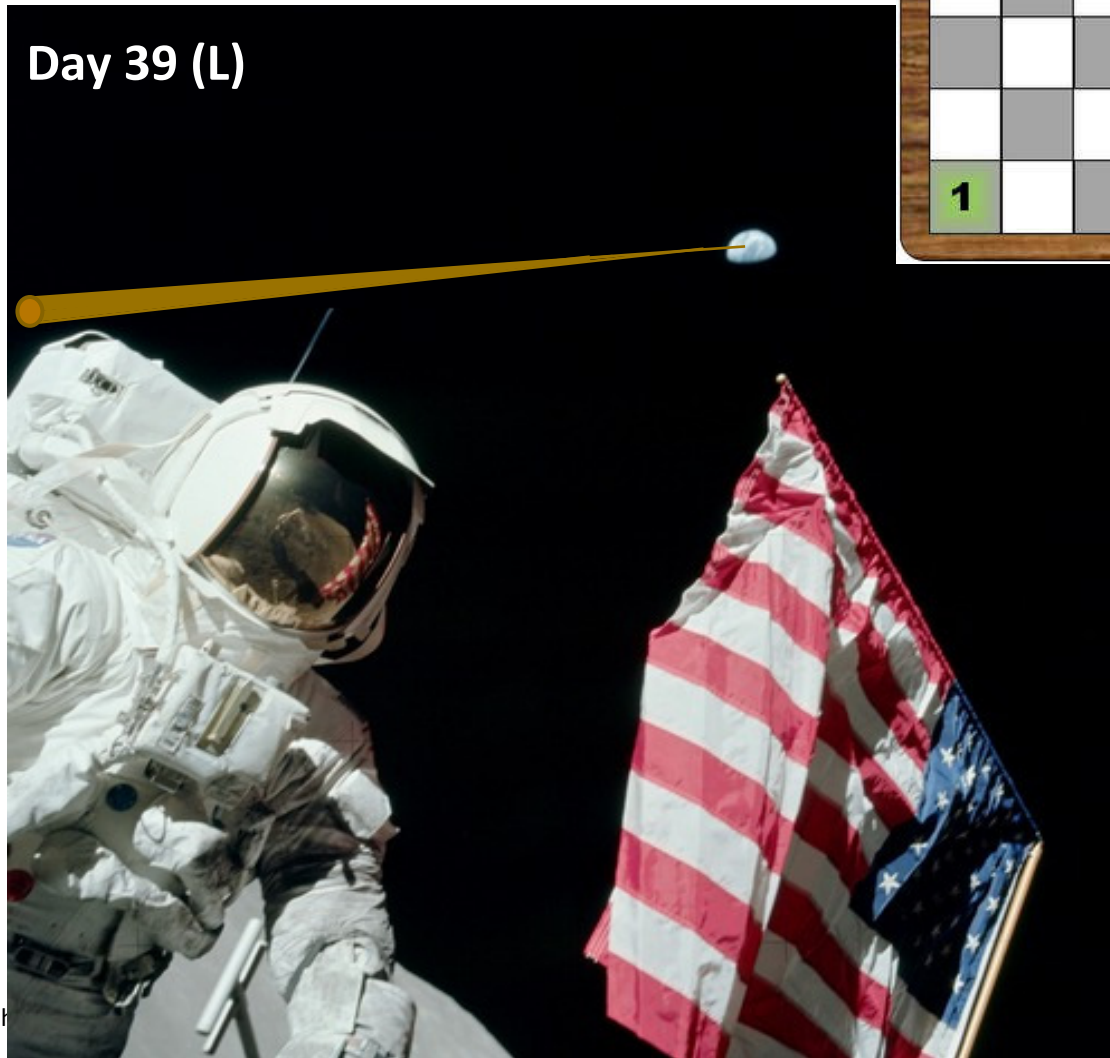
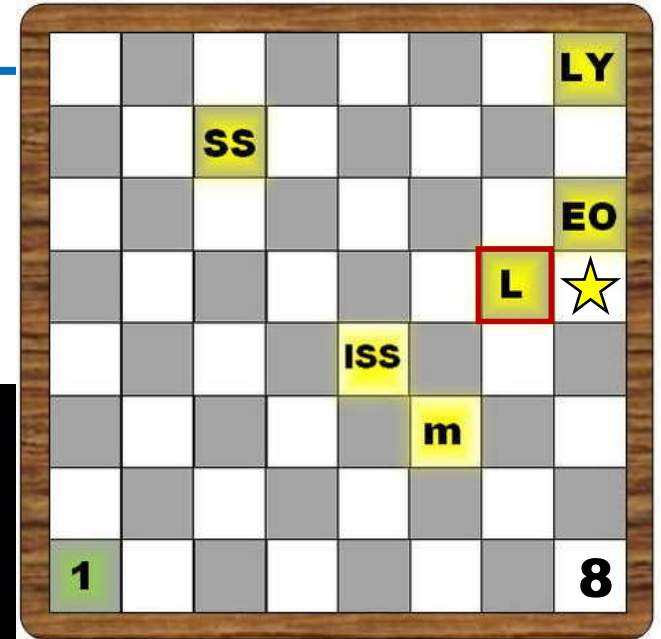
*Tomorrow's stack will go twice as far.*



# The CM's story - 2

**Day 39** Imagine the surprise at our Lunar Colony base as the penny spike shoots past the moon!

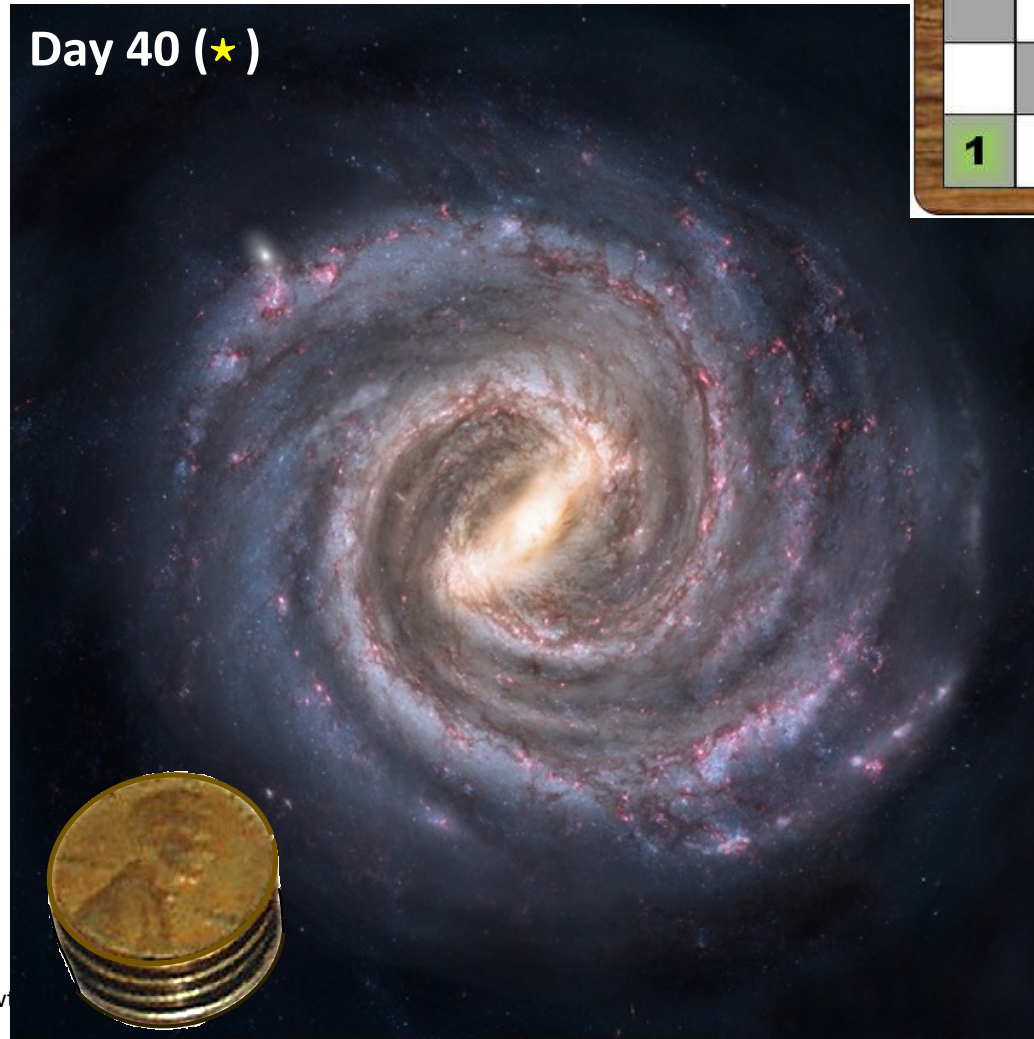
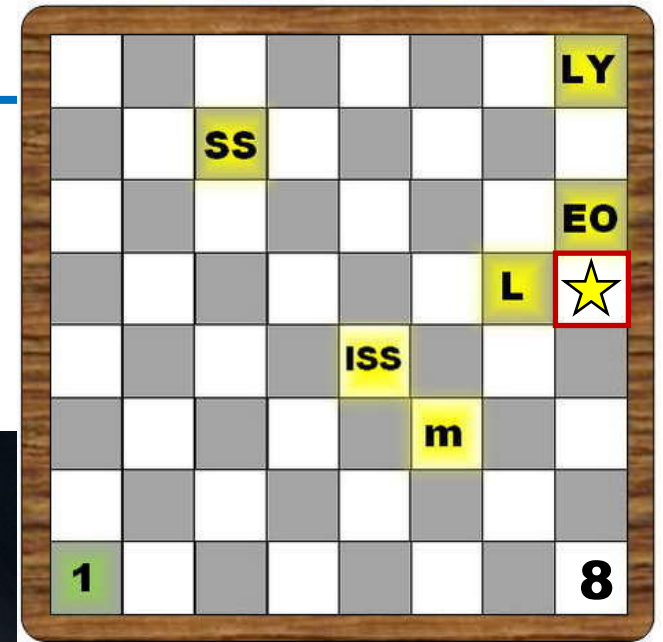
*Tomorrow's stack will go twice as far.*



# The CM's story - 3

**Day 40** The pennies in the stack out number the twinkling stars in the Milky Way ( $200-400 \times 10^9$ )

*Tomorrow's stack will go twice as far.*



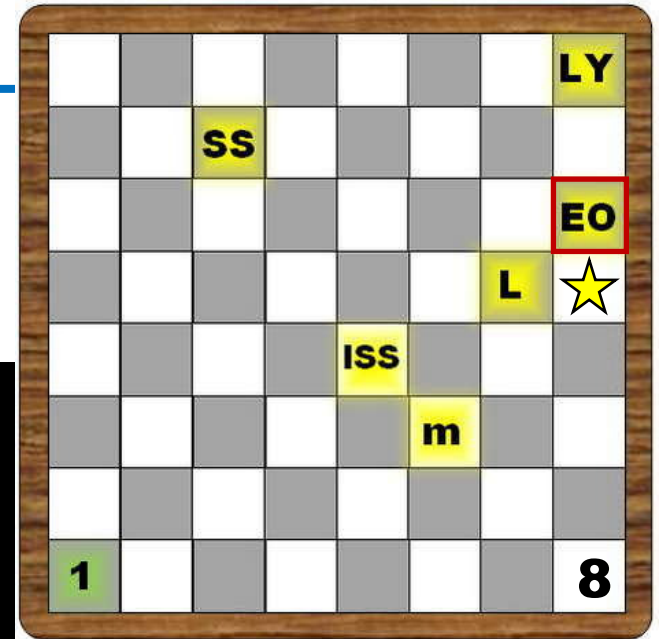
Day 40 (★)



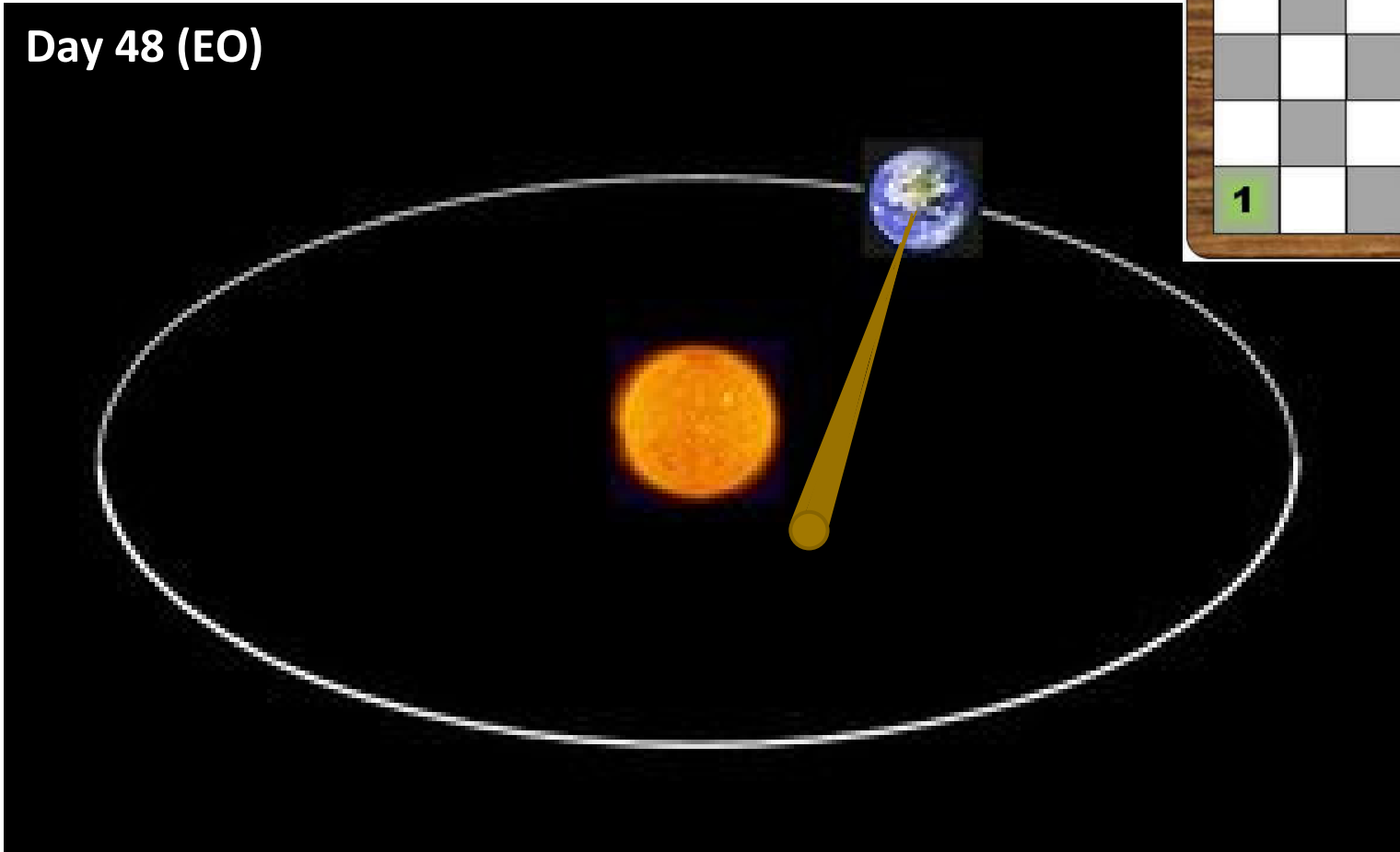
# The CM's story – 4

**Day 48** Length of the penny stack exceeds Earth-Sun distance (93 M mi, 150 M km).

*Tomorrow > diameter of the Earth's orbit*



**Day 48 (EO)**

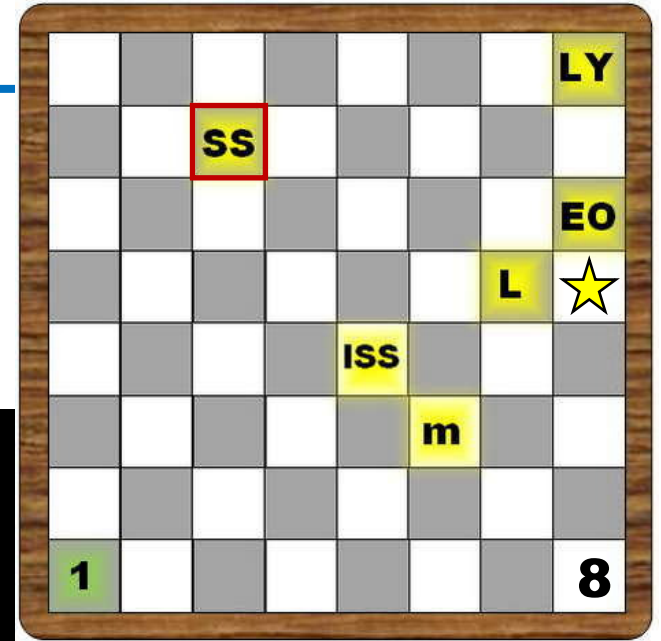


*Tomorrow's stack will go twice as far.*

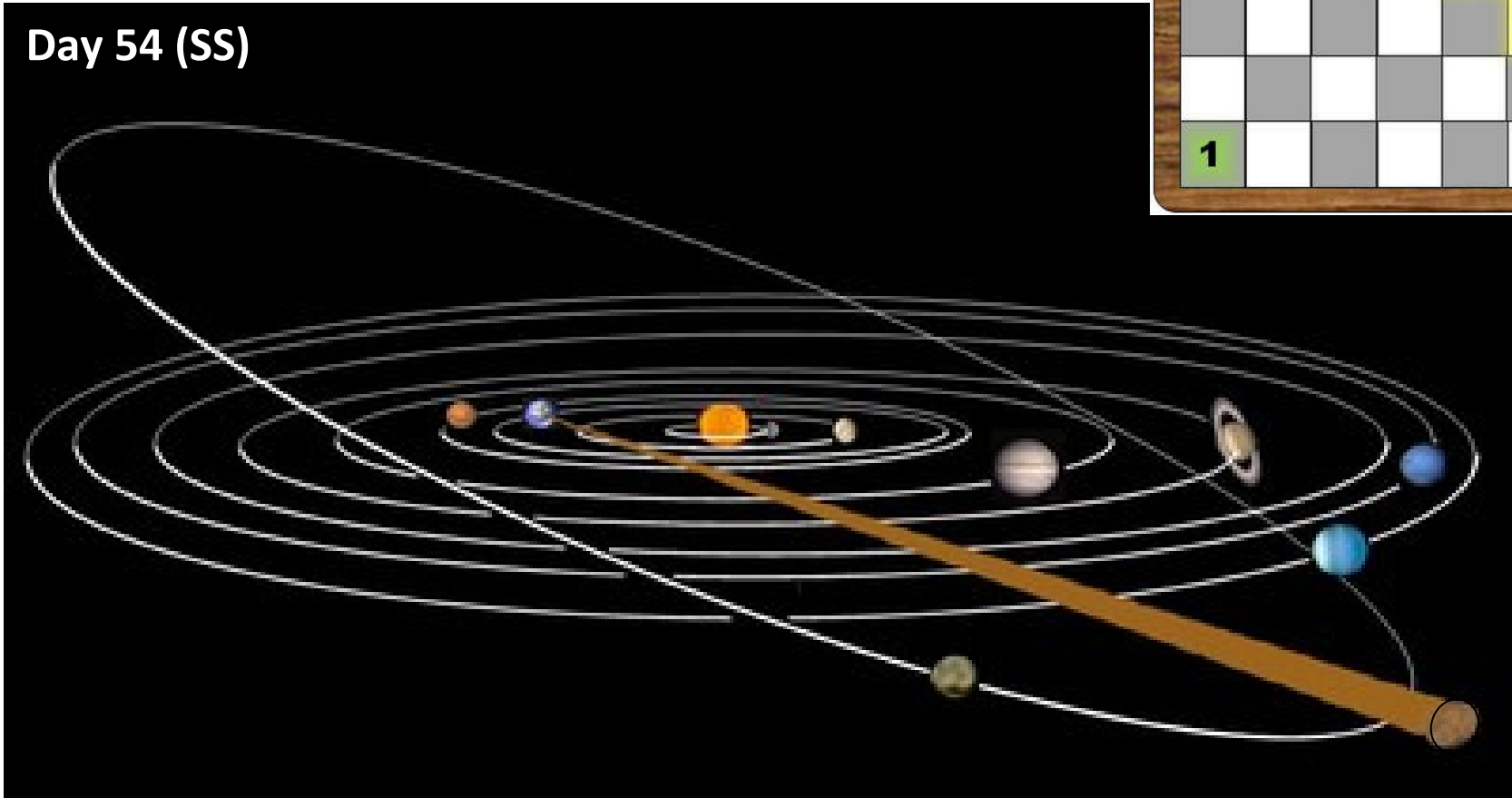
# The CM's story – 5

Day 54 Penny stack now longer than the *radius* of the solar system. 7.5 billion miles or  $12 \times 10^9$  km

*Tomorrow's stack exceeds diameter of solar system.*



Day 54 (SS)



*Tomorrow's stack will always go twice as far.*

# The end of the CM's story – 6

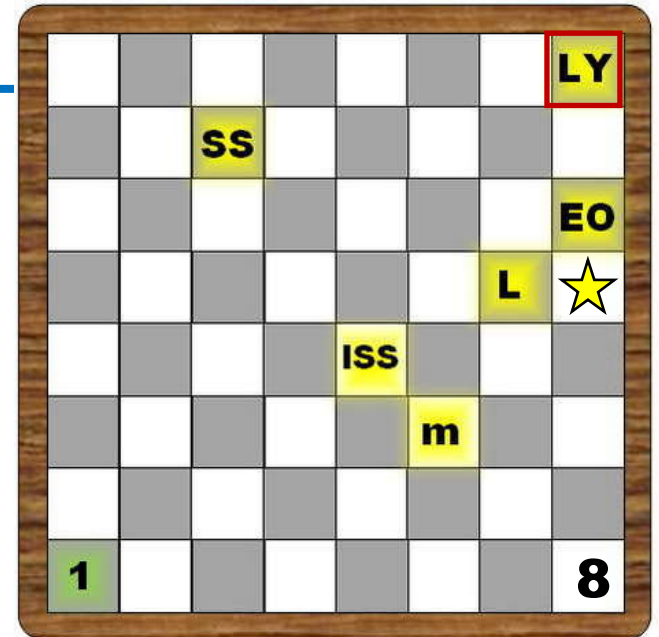
**CM:** "true, sustainability might become an issue?  
but *what if* we continued ... "

to the very end

**Day 64** Stack > 1 light year.

and just 2 days longer

**Day 66** ...*at Alpha Centauri*



*Each and every inevitable day, exponential growth in the penny stack  
Tomorrow's stack of pennies must always go twice as far as today's*

# Aside from inventing warp drive ...

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## ▶ **The King clearly had a couple bad months**

Several endings:

- King gives in to the inevitable – BYM, Princess marries
- King is full of rage – orders BYM executed (with consequences)
- ... there are other endings, too.

## ▶ **This is story of unconstrained 'explosive' growth**

- Coin stack starts slowly, no problem – grows unbelievably fast
- Rapid growth – poses impossible logistics problem (physics truths aside)
  - Unbelievable challenge to ever assemble daily coin stack
  - Impossible growth demand on copper supply-line
  - Relentless expansion of facility needs

**Most growth** begins with free, unconstrained expansion  
winds up in non-exponential *highly constrained* situation.